FINITE-ELEMENT ANALYSIS OF MACHINES

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Summary

With the advent of improved computing power and numerical methods in recent years, it is now practical to use *finite-element models* (FEM) to compute the magnetic fields in electrical machines. The time-stepping FEM, in particular, enables the designer to solve problems that are difficult to consider when using the analytical approach. Many empirical factors no longer need to be considered. Consequently, computed performance figures are becoming much more accurate. This is particularly useful for design optimization on induction motors, which have already been well developed using the circuit model.

The aim of the article is to provide a review of the application of FEM in the study of induction motors. Different FEMs, involving a 2-D magnetostatic model, a 2-D complex eddy-current model, a 2-D rotating model, a 2-D FEM coupled with circuit equations model, and a 3-D model, are all discussed. In addition, it is possible to use a multi-slice time-stepping 2-D model to study an essentially 3-D machine phenomenon, such as the effect of skewing of the rotor bars in an induction motor. The effects of the inverters upon the FEM are also discussed, and the merits and limitations of the various FEM models are given.

1. Introduction

Induction motors have been, and will continue to be, the main workhorses of industry in the foreseeable future. Due to the keen competition that exists among motor manufacturers, end-users are expecting better and better performance from motors. The conventional equivalent circuits used to analyze and design induction motors are no longer accurate enough for many purposes.

With the use of FEM, one can study the actual flux distribution from a given current distribution. Hence the correct saturation of different parts, the use of magnetic wedges, and the influence of shaft loads can readily be accounted for. The "skin effect" in rotor bars, an understanding of which is essential in analyzing the starting behavior, can also be investigated without relying on overall correction factors, which are normally required by analytical procedures.

1.1. Brief Review of FEM

Although the development of numerical computation in the study of electromagnetic fields dates back only about 30 years, it is now maturing into an independent subject area with high commercial potential. Among the various numerical methods, FEM has a dominant position because it is versatile, has considerable interchangeability, and can be incorporated readily into standard programs.

In 1973 Chari pioneered the application of FEM in the computation of electromagnetic fields, and subsequent data reaffirmed the importance of FEM in their numerical study. The application of FEM in the computation of electromagnetic fields has been extended steadily from linear to non-linear methods, from static field to alternating field and transient field, and from 2-D to 3-D. Moreover, the inclusion of pulse-width modulation (PWM) in inverters is adding further complexity to FEM models. Nonetheless, a lot of developmental effort has gone into FEM, and the computation power of these algorithms has been increasing rapidly in order to satisfy the demand for high accuracy, short computation time, and small memory requirements.

Indeed, the study of 2-D non-linear alternating fields in the frequency domain has already matured to the extent that it is widely accepted in industry. The most complete model of 2-D field analysis involves using a time-stepping FEM, and these approaches date back to the 1980s. The time-stepping method can deal with transient problems, and can also be used to calculate steady-state fields when current and magnetic waveforms are non-sinusoidal. When circuit and mechanical equations are coupled with the FEM, currents and torque can be evaluated readily without any iteration.

At present, 3-D eddy-current analysis still presents a big challenge to electrical machine designers. Because of the complicated mathematical models involved and the need for much CPU time and storage space, coupled with the requirement for pre-processors and post-processors, the 3-D field is much more difficult to handle than its 2-D counterpart. Thus 3-D applications are still confined to simplified models only.

For solving large FEM equations, methods are developed from Gaussian elimination, followed by the triangular factorization to the incomplete Cholesky-conjugate gradient (ICCG) algorithm and shifting ICCG algorithm. ICCG has the advantages of being fast and requiring small storage resources. In dealing with more general equations whose coefficient matrix is asymmetrical a bi-conjugate gradient (BICG) method, with suitable preconditioning to enhance the convergence speed, must be used. For non-linear problems, the Newton–Raphson technique is usually employed. Several researchers have presented accelerated Newton–Raphson techniques; they have also reported that it is possible to reduce the CPU time required to as much as 50–60% of that required by standard methods.

In studying the performance of induction motors, FEM modeling can be divided into two areas:

- equivalent circuit–magnetic field method
- external circuit–electromagnetic field method.

In the *equivalent circuit–magnetic field method*, the simulation is based on the traditional equivalent circuit approach. FEM, as a complement to the circuit method, is used to compute the circuit parameters. The stator and rotor currents are calculated outside the field solution. With this method computation time is relatively short, although its accuracy is limited by the circuit concept.

In the *external circuit–electromagnetic field method*, the simulation is based on the field concept. The behavior of the machine is determined directly by the distribution of magnetic fields and current density. The stator currents and rotor current densities can be computed concurrently, and skin effect can then be taken into account. The disadvantage is the additional computation time required.

For the sake of clarity and simplicity, the following description will focus mainly on the different mathematical models used in various electromagnetic field computations.

2. 2-D Magnetostatic Model

In 2-D analysis, the end-effects are neglected. The vector potential consists only of an axial component. The magnetic field is present in planes normal to the machine axis. The transverse fields of the machines are studied. With the field source being assumed as constant, with no induced eddy-currents, one obtains:

$$\frac{\partial}{\partial x}\left(\nu\frac{\partial A}{\partial x}\right)+\frac{\partial}{\partial y}\left(\nu\frac{\partial A}{\partial y}\right)=-J \qquad \text{in region } \Omega$$

$$A=A_{z0} \text{ on first boundary condition } S_1 \qquad (1)$$

$$\frac{\partial A}{\partial n}=-\frac{1}{\nu}H_t \text{ on second boundary condition } S_2$$

where A is the vector potential which coincides with its component A_z, ν is the magnetic reluctivity, J is the current density, A_{z0} is the known value of A on first boundary condition, and H_t is the known value of the tangential component of the magnetic field intensity on second boundary condition. By using FEM, the boundary value problem is equivalent to the following problem:

$$W(A) = \iint_{\Omega}\left[\int_0^B H\mathrm{d}B - JA\right]\mathrm{d}\Omega + \int_{S_2} H_t A\mathrm{d}S = \min \tag{2}$$

where B is the magnetic flux density and H is the magnetic field strength. By discretization, the matrix equation can be written as:

$$[C]\ [A] = [P] \tag{3}$$

where $[C]$ is the coefficient matrix and $[P]$ is the vector associated with input currents. From Eq. (3) one obtains the magnetic vector potentials in the regions. With this model, one could calculate the stator air-gap factor and study the relationship of the slot-leakage reactance upon magnetic, non-magnetic, and anisotropic magnetic slot wedges. Moreover, this model could be used to compute and express the high-order space harmonics in the air-gap. Hence the influence of the anisotropy of magnetic materials and higher-order harmonics due to saturation can be considered. Slot-harmonic effects in induction motors could also be studied with the assistance of a series of linear magnetostatic field solutions in the stator-rotor inductance matrix. This model has also been used to simulate the no-load test, and will be discussed in more detail in the next section.

Note also that the 2-D magnetostatic model cannot describe the exact performance of induction motors because the eddy-current, the rotor movement, and the non-sinusoidal quantities cannot be taken into account. The model is limited to the calculation of parameters in the equivalent circuit, and the study of some local phenomena that are hard to obtain using analytical approaches.

3. 2-D Complex Eddy-Current Model

When eddy-currents are considered, the Maxwell equations can be applied to the 2-D domains to give the following equation:

$$\frac{\partial}{\partial x}\left(\nu\frac{\partial A}{\partial x}\right) + \frac{\partial}{\partial y}\left(\nu\frac{\partial A}{\partial y}\right) = -J + \sigma\frac{\partial A}{\partial t} \tag{4}$$

where σ is the conductivity of material. If the model is linear, the magnetic source and the magnetic field will change sinusoidally with time. Hence J and A are sinusoidal vectors, and $\partial A/\partial t$ can be represented by $j\omega A$. Moreover,

$$W(A) = \iint_{\Omega}\left[\int_0^B H\mathrm{d}B - JA + \frac{1}{2}j\omega\sigma A^2\right]\mathrm{d}\Omega + \int_{S_2} H_t A\mathrm{d}S = \min \tag{5}$$

If the model is non-linear, it is necessary to determine the equivalent reluctivity of the materials. Different definitions of equivalent reluctivity have been summarized by Williamson.

The "standard" domain to be investigated spans one pole, or pair of poles: this is the smallest sector of the motor, containing an integral number of slots or teeth in both stator and rotor. This arrangement ensures that symmetry and periodicity are strongly enforced in the model. If the fields of stator and rotor are computed separately, a phase-band stator model and a single-slot rotor model can also be used. The advantage of choosing these reduced domains is the reduction in computation time. Their disadvantages are that the asymmetrical features cannot be represented, and the method is limited to balanced polyphase machines.

The static model and the complex model have been used to simulate the no-load test and the locked-rotor test in order to compute the parameters in the equivalent circuit. For the no-load situation, a static solution is used because no currents are induced in the rotor bars, as the rotor is assumed to be rotating at synchronous speed. The no-load inductance is found by equating the magnetic energy stored in the equivalent circuit to the magnetic energy in the FEM model. Using the loss curve of the iron supplied by the manufacturer, the iron losses could be found. The no-load conductance of the equivalent circuit can then be computed. For the locked-rotor test, a complex eddy-current model is used to give the machine parameters, both at standstill and at rated voltage. The complex power resulting from the FEM model is used to estimate the parameters.

With the complex model, one then calculates the starting current and the characteristics on entering the locked-rotor condition. Williamson also describes a technique for calculating the leakage reactance with closed slots, taking only one tooth-pitch of the rotor as the solution region. Moreover, Williamson has further refined his method, which includes the effects of rotor harmonics and rotor skewing, with the aid of circuit equations. Other workers also propose coupling FEM with a circuit model to calculate the harmonic losses in alternating-current (AC) machine windings at different frequencies.

The precision of this model is also limited by the concept of the equivalent circuit. The rotation of the rotor and non-sinusoidal quantities cannot be taken into account, even if the eddy-current has been included.

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Biographical Sketch

Siu-Lau Ho received B.Sc. and Ph.D. degrees in electrical engineering from the University of Warwick in 1976 and 1979, respectively. Since 1979, Dr. Ho has been with the Department of Electrical Engineering, Hong Kong Polytechnic University, where he is now a Professor. His current research interests include condition monitoring and thermal characteristics of electrical machines, finite-element analysis, and developing meshless methods for design optimization. Professor Ho is a Chartered Engineer, a member of the UK Institution of Electrical Engineers, and a member of the Hong Kong Institution of Engineers.